Gravito-Electromagnetic coupled perturbations and QNMs of a charged black hole with scalar hair

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Outline

- 1. Introduction
- 2. Perturbation equations
- 3. Quasinormal modes
- 4. Conclusions

1. Introduction

Singularity? Topological star/black hole model [1,2,3]

A five-dimensional Einstein-Maxwell theory

$$S = \int d^5x \sqrt{-\hat{g}} \left(\frac{1}{16\pi G_5} \hat{R} - \frac{1}{16\pi} \hat{F}^{MN} \hat{F}_{MN} \right)$$

The extra dimension y is a warped circle with radius R_y

$$\begin{aligned} ds^2 &= -f_S(r)dt^2 + f_B(r)dy^2 + \frac{1}{f_S(r)f_B(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \\ F &= P \sin\theta d\theta \wedge d\phi, \end{aligned}$$

$$f_B(r) = 1 - \frac{r_B}{r} \qquad f_S(r) = 1 - \frac{r_S}{r} \qquad P = \pm\frac{1}{G_5}\sqrt{3 r_S r_B}$$

Similar to the classical black hole in macrostate geometries

Constructed from type IIB string theory

[1] I. Bah and P. Heidmann, Phys. Rev. Lett. 126, 151101 (2021), [2011.08851].
[2] I. Bah and P. Heidmann, [2012.13407].
[3] S. Stotyn and R. B. Mann, Phys. Lett. B 705, 269 (2011), [1105.1854].

1. Introduction

KK reduction

A four-dimensional Einstein-Maxwell-dilaton theory

$$S_4 = \int d^4x\sqrt{-g}\Big(\frac{1}{16\pi G_4}R_4 - \frac{3}{8\pi G_4}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{e^{-2\Phi}}{16\pi e^2}F_{\mu\nu}F^{\mu\nu}\Big).$$

$$G_4 = e^2 G_5 \qquad e^2 \equiv \frac{1}{2\pi R_y} \qquad e^{2\Phi} = f_B^{-1/2}$$

Metric $\quad ds_4^2 = f_B^{\frac{1}{2}}[-f_S dt^2 + \frac{dr^2}{f_B f_S} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2]$

$$f_B(r) = 1 - \frac{r_B}{r}$$

$r < r_B \quad f_B^{1/2}$ becomes to imaginary

$r = r_B$ is the end of the spacetime

A four-dimensional static spherical symmetric charged black hole with scalar hair

1. Introduction

Electromagnetic field $A_\mu = (0,0,0,-\frac{e}{2}\sqrt{\frac{3r_B r_S}{G_4}}\cos\theta)$

Mass and magnetic charge

$$M = \left(\frac{2r_S + r_B}{4G_4}\right),$$
$$Q_{\rm m} = \frac{1}{2}\sqrt{\frac{3r_B r_S}{G_4}}.$$

In terms of M and Q_m

$$r_S^{(1)} = 2G_4(M - M_\triangle), \qquad r_B^{(1)} = G_4(M + M_\triangle),$$
$$r_S^{(2)} = G_4(M + M_\triangle), \qquad r_B^{(2)} = 2G_4(M - M_\triangle).$$

1. Introduction

where $$M_{\triangle}^2 = M^2 - \left(\frac{\sqrt{2}Q_{\mathrm{m}}}{\sqrt{3G_4}}\right)^2$$

NS $$0 \leq Q_m \leq \sqrt{\frac{3}{2}} G_4 M$$ larger

RN $$0 \leq Q \leq G_4 M$$

Gregory-Laflamme instability [4]?

The stability of small perturbations

$$r_S^{(2)} = G_4(M + M_{\triangle}), \qquad r_B^{(2)} = 2G_4(M - M_{\triangle}).$$

The second solution is stable [3].

[3] S. Stotyn and R. B. Mann, Phys. Lett. B 705, 269 (2011), [1105.1854].
[4] R. Gregory and R. Laflamme, Phys. Rev. Lett. 70, 2837 (1993), [hep-th/9301052].

2. Perturbation equations

Perturbed fields

$$
\begin{aligned}
\Phi &= \bar{\Phi} + \varphi, \\
A_\mu &= \bar{A}_\mu + a_\mu, \\
g_{\mu\nu} &= \bar{g}_{\mu\nu} + h_{\mu\nu}
\end{aligned}
$$

background

perturbations

spherical symmetric

spherical harmonic

scalars, two-dimensional vectors, and two-dimensional tensors

Scalar base $Y_{l,m}(\theta,\phi)$

Vector bases

$$(V^1_{l,m})_a = \partial_a Y_{l,m}(\theta,\phi),$$
$$(V^2_{l,m})_a = \gamma^{bc}\epsilon_{ac}\partial_b Y_{l,m}(\theta,\phi).$$

Tensor bases

$$(T^1_{l,m})_{ab} = (Y_{l,m})_{;ab}\,,$$
$$(T^2_{l,m})_{ab} = Y_{l,m}\gamma_{ab},$$
$$(T^3_{l,m})_{ab} = \frac{1}{2}\left[\epsilon^c_{\ a}(Y_{l,m})_{;cb} + \epsilon^c_{\ b}(Y_{l,m})_{;ca}\right],$$

Space inversion

Even or polar parity	$(-1)^l$	$Y_{l,m}, V^1_{l,m}, T^1_{l,m}, T^2_{l,m}$
Odd or axial parity	$(-1)^{l+1}$	$V^2_{l,m}, T^3_{l,m}$

2. Perturbation equations

Background scalar field and metric field are even parity

$$e^{2\Phi} = f_B^{-1/2}$$

$$ds_4^2 = f_B^{\frac{1}{2}}[-f_S dt^2 + \frac{dr^2}{f_B f_S} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2]$$

Background magnetic field is odd parity

$$A_\mu = (0, 0, 0, -\frac{e}{2}\sqrt{\frac{3r_B r_S}{G_4}}\cos\theta)$$

Scalar perturbation and even-parity parts of the metric perturbations couple to the odd-parity parts of the electromagnetic perturbations (type-I coupling)

The odd-parity parts of the metric perturbations couple to the even-parity parts of the electromagnetic perturbations (type-II coupling)

2. Perturbation equations

In the Regge-Wheeler gauge [5]

Odd parts of the perturbation $h_{\mu\nu}$

$$h_{\mu\nu} = \sum_l e^{-i\omega t} \begin{bmatrix} 0 & 0 & 0 & h_0 \\ 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix} \sin\theta \partial_\theta Y_{l,0}(\theta)$$

Following ref. [6]

$$\tilde{f}_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu,$$

even parts of the perturbation $\tilde{f}_{\mu\nu}$

$$\tilde{f}_{\mu\nu} = \sum_l e^{-i\omega t} \begin{bmatrix} 0 & f_{01} & f_{02} & 0 \\ * & 0 & f_{12} & 0 \\ 0 & * & 0 & 0 \\ 0 & * & 0 & 0 \end{bmatrix} \sin\theta \partial_\theta Y_{l,0}(\theta)$$

we have chosen $m = 0$ for simplify

[5] T. Regge and J. A. Wheeler, Phys. Rev. 108, 1063 (1957).
[6] F. J. Zerilli, Phys. Rev. D 9, 860-868 (1974).

2. Perturbation equations

Equivalent to

$$
\begin{aligned}
a_t &= -\sum_l e^{-i\omega t} f_{02} Y_{l,0}, \\
a_r &= -\sum_l e^{-i\omega t} f_{12} Y_{l,0}, \\
a_\theta &= 0, \\
a_\phi &= 0.
\end{aligned}
$$

$$\tilde{f}_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad \Longrightarrow \quad f_{01} = \partial_r f_{02} + i\omega f_{12}$$

Tortoise coordinate

$$dr_* = \frac{1}{\sqrt{f_B f_S}} dr$$

2. Perturbation equations

Master perturbation equations

$$\frac{d^2\psi_g}{dr_*^2} + (\omega^2 - V_{11})\psi_g - V_{12}\psi_m = 0$$
$$\frac{d^2\psi_m}{dr_*^2} + (\omega^2 - V_{22})\psi_m - V_{21}\psi_g = 0$$

where

$$V_{11} = f_S\left[\frac{l(l+1)}{r^2} - \frac{3(r_B^2(13r_S - 9r) + 16r_S r^2)}{16 f_B r^5}\right] + f_S\frac{3r_B(2r - 7r_S)}{4 f_B r^4},$$

$$V_{12} = -\frac{i f_S f_B^{1/4}}{e l(l+1) r^3}\sqrt{6 r_B r_S}\,\omega\kappa_4,$$

$$V_{21} = \frac{i\sqrt{\frac{3}{2} r_B r_S}\, e f_S}{\kappa_4 \omega f_B^{1/4} r^3}(l-1)l(l+1)(l+2),$$

$$V_{22} = f_S\left[\frac{3r_B r_S}{r^4} + \frac{l(l+1)}{r^2}\right].$$

$$f_B(r) = 1 - \frac{r_B}{r}$$

$$\tilde{\psi}_m \equiv \frac{\psi_m}{e}$$

eliminate e

3. Quasinormal modes

Matrix-valued direct integration method [7]

Compact form

$$\frac{d^2\mathbf{Y}}{dr_*^2} + (\omega^2 - \mathbf{V})\mathbf{Y} = 0$$

$$\mathbf{Y} = \begin{pmatrix} \psi_g \\ \psi_m \end{pmatrix}$$

Boundary conditions

Pure ingoing waves at the event horizon

$$Y_i \sim b_i e^{-i\omega r_*}, \quad r_* \to -\infty,$$

Pure outgoing waves at spatial infinity

$$Y_i \sim B_i e^{i\omega r_*}, \quad r_* \to +\infty$$

[7] P. Pani, Int. J. Mod. Phys. A 28, 1340018 (2013)

3. Quasinormal modes

Matrix-valued continued fraction method [8]

Eigenfunctions

$$\psi_{\rm g} = (r-r_S)^{-p}(r-r_S+1)^p e^{i(r-r_S)\omega}(r-r_S+1)^{i(r_B/2+r_S)\omega}\sum_n a_n^g H(r)^n,$$

$$\psi_{\rm m} = (r-r_S)^{-p}(r-r_S+1)^p e^{i(r-r_S)\omega}(r-r_S+1)^{i(r_B/2+r_S)\omega} f_B(r)^{3/4}\sum_n a_n^m H(r)^n,$$

where $p = \frac{i r_S^{3/2}\omega}{\sqrt{r_s - r_B}}$ $H(r) = \frac{r-r_S}{r-r_B}$

[8] E. W. Leaver, Proc. R. Soc. Lond. A 402, 285-298 (1985).

3. Quasinormal modes

seven-term recurrence relations

$$
\begin{aligned}
\alpha_0 \mathbf{A_1} &+ \beta_0 \mathbf{A_0} = 0, \\
\alpha_1 \mathbf{A_2} &+ \beta_1 \mathbf{A_1} + \gamma_1 \mathbf{A_0} = 0, \\
\alpha_2 \mathbf{A_3} &+ \beta_2 \mathbf{A_2} + \gamma_2 \mathbf{A_1} + \rho_2 \mathbf{A_0} = 0, \\
\alpha_3 \mathbf{A_4} &+ \beta_3 \mathbf{A_3} + \gamma_3 \mathbf{A_2} + \rho_3 \mathbf{A_1} + \lambda_3 \mathbf{A_0} = 0, \\
\alpha_4 \mathbf{A_5} &+ \beta_4 \mathbf{A_4} + \gamma_4 \mathbf{A_3} + \rho_4 \mathbf{A_2} + \lambda_4 \mathbf{A_1} \\
&+ \sigma_4 \mathbf{A_0} = 0, \\
\alpha_{\mathbf{n}} \mathbf{A_{n+1}} &+ \beta_{\mathbf{n}} \mathbf{A_n} + \gamma_{\mathbf{n}} \mathbf{A_{n-1}} + \rho_{\mathbf{n}} \mathbf{A_{n-2}} + \lambda_{\mathbf{n}} \mathbf{A_{n-3}} \\
&+ \sigma_{\mathbf{n}} \mathbf{A_{n-4}} + \delta_{\mathbf{n}} \mathbf{A_{n-5}} = 0,
\end{aligned}
$$

$$
\mathbf{A_n} = \begin{pmatrix} a_n^g \\ a_n^m \end{pmatrix}
$$

A three-term recurrence relation can be obtained through a matrix-valued version of the Gaussian elimination

Solve the inverse of the coefficient matrices

3. Quasinormal modes

$$
\begin{pmatrix}
\beta_0^{11} & \beta_0^{12} & \alpha_0^{11} & \alpha_0^{12} & & & & & & & & & & & & \\
\beta_0^{21} & \beta_0^{22} & \alpha_0^{21} & \alpha_0^{22} & & & & & & & & & & & & \\
\gamma_1^{11} & \gamma_1^{12} & \beta_1^{11} & \beta_1^{12} & \alpha_1^{11} & \alpha_1^{12} & & & & & & & & & & \\
\gamma_1^{21} & \gamma_1^{22} & \beta_1^{21} & \beta_1^{22} & \alpha_1^{21} & \alpha_1^{22} & & & & & & & & & & \\
\rho_2^{11} & \rho_2^{12} & \gamma_2^{11} & \gamma_2^{12} & \beta_2^{11} & \beta_2^{12} & \alpha_2^{11} & \alpha_2^{12} & & & & & & & & \\
\rho_2^{21} & \rho_2^{22} & \gamma_2^{21} & \gamma_2^{22} & \beta_2^{21} & \beta_2^{22} & \alpha_2^{21} & \alpha_2^{22} & & & & & & & & \\
\lambda_3^{11} & \lambda_3^{12} & \rho_3^{11} & \rho_3^{12} & \gamma_3^{11} & \gamma_3^{12} & \beta_3^{11} & \beta_3^{12} & \alpha_3^{11} & \alpha_3^{12} & & & & & & \\
\lambda_3^{21} & \lambda_3^{22} & \rho_3^{21} & \rho_3^{22} & \gamma_3^{21} & \gamma_3^{22} & \beta_3^{21} & \beta_3^{22} & \alpha_3^{21} & \alpha_3^{22} & & & & & & \\
\sigma_4^{11} & \sigma_4^{12} & \lambda_4^{11} & \lambda_4^{12} & \rho_4^{11} & \rho_4^{12} & \gamma_4^{11} & \gamma_4^{12} & \beta_4^{11} & \beta_4^{12} & \alpha_4^{11} & \alpha_4^{12} & & & & \\
\sigma_4^{21} & \sigma_4^{22} & \lambda_4^{21} & \lambda_4^{22} & \rho_4^{21} & \rho_4^{22} & \gamma_4^{21} & \gamma_4^{22} & \beta_4^{21} & \beta_4^{22} & \alpha_4^{21} & \alpha_4^{22} & & & & \\
\delta_5^{11} & \delta_5^{12} & \sigma_5^{11} & \sigma_5^{12} & \lambda_5^{11} & \lambda_5^{12} & \rho_5^{11} & \rho_5^{12} & \gamma_5^{11} & \gamma_5^{12} & \beta_5^{11} & \beta_5^{12} & \alpha_5^{11} & \alpha_5^{12} & & \\
\delta_5^{21} & \delta_5^{22} & \sigma_5^{21} & \sigma_5^{22} & \lambda_5^{21} & \lambda_5^{22} & \rho_5^{21} & \rho_5^{22} & \gamma_5^{21} & \gamma_5^{22} & \beta_5^{21} & \beta_5^{22} & \alpha_5^{21} & \alpha_5^{22} & & \\
 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\
 & & \delta_n^{11} & \delta_n^{12} & \sigma_n^{11} & \sigma_n^{12} & \lambda_n^{11} & \lambda_n^{12} & \rho_n^{11} & \rho_n^{12} & \gamma_n^{11} & \gamma_n^{12} & \beta_n^{11} & \beta_n^{12} & \alpha_n^{11} & \alpha_n^{12} \\
 & & \delta_n^{21} & \delta_n^{22} & \sigma_n^{21} & \sigma_n^{22} & \lambda_n^{21} & \lambda_n^{22} & \rho_n^{21} & \rho_n^{22} & \gamma_n^{21} & \gamma_n^{22} & \beta_n^{21} & \beta_n^{22} & \alpha_n^{21} & \alpha_n^{22}
\end{pmatrix}
\begin{pmatrix}
a_0^g \\ a_0^m \\ a_1^g \\ a_1^m \\ a_2^g \\ a_2^m \\ a_3^g \\ a_3^m \\ a_4^g \\ a_4^m \\ a_5^g \\ a_5^m \\ \vdots \\ a_n^g \\ a_n^m
\end{pmatrix}
= 0.
$$

The QNMs are those which make the determinant of the coefficient matrix is zero.

3. Quasinormal modes

$Q_{\rm m}/M$	Charged BH DI		Charged BH CF		Q/M	RN BH	
	$\omega_{\rm R}M$	$\omega_{\rm I}M$	$\omega_{\rm R}M$	$\omega_{\rm I}M$		$\omega_{\rm R}M$	$\omega_{\rm I}M$
0	0.37367	-0.088962	0.37367	-0.088962	0	0.37367	-0.088962
0.2	0.37474	-0.089081	0.37480	-0.089095	0.2	0.37474	-0.089075
0.4	0.37848	-0.089429	0.37855	-0.089463	0.4	0.37844	-0.089398
0.6	0.38641	-0.089982	0.38649	-0.090086	0.6	0.38622	-0.089814
0.8	0.40163	-0.090500	0.40169	-0.090886	0.8	0.40122	-0.089643
1.12	0.47027	-0.084231	0.47153	-0.092731	0.9999	0.43134 [77]	-0.083460 [77]

fundamental QNMs for the gravitational field ψ_g with $l = 2$

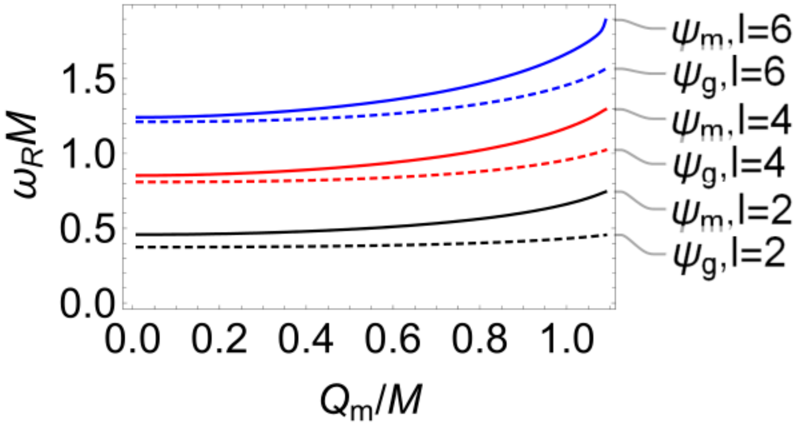
3. Quasinormal modes

Q_m/M	Charged BH DI		Charged BH CF		Q/M	RN BH	
	$\omega_R M$	$\omega_I M$	$\omega_R M$	$\omega_I M$		$\omega_R M$	$\omega_I M$
0	0.45715	-0.094784	0.45715	-0.094784	0	0.45759	-0.095004
0.2	0.46295	-0.095377	0.46296	-0.095359	0.2	0.46297	-0.095373
0.4	0.47969	-0.096462	0.47969	-0.096441	0.4	0.47993	-0.096442
0.6	0.51053	-0.098155	0.51055	-0.098133	0.6	0.51201	-0.098017
0.8	0.56316	-0.10008	0.56320	-0.10002	0.8	0.57013	-0.099069
1.12	0.78258	-0.091135	0.79925	-0.098085	0.9999	0.70430 [77]	-0.085973 [77]

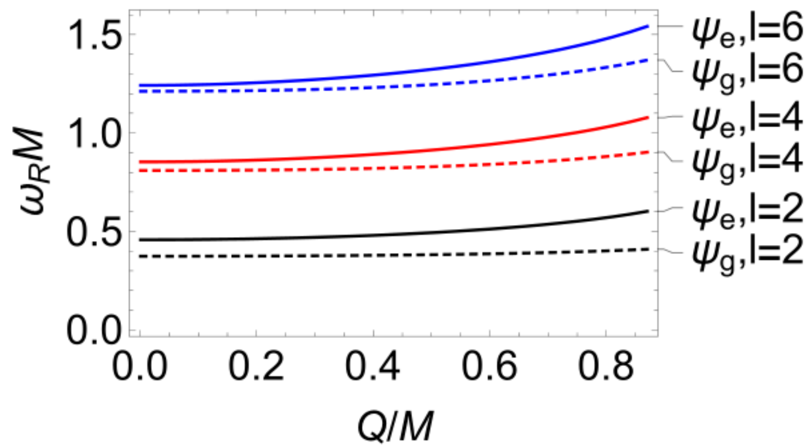
fundamental QNMs for the magnetic field ψ_m, with $l = 2$

The differences of the frequencies of fundamental QNMs between the charged black hole with a scalar hair and the RN balck hole are very small.

3. Quasinormal modes

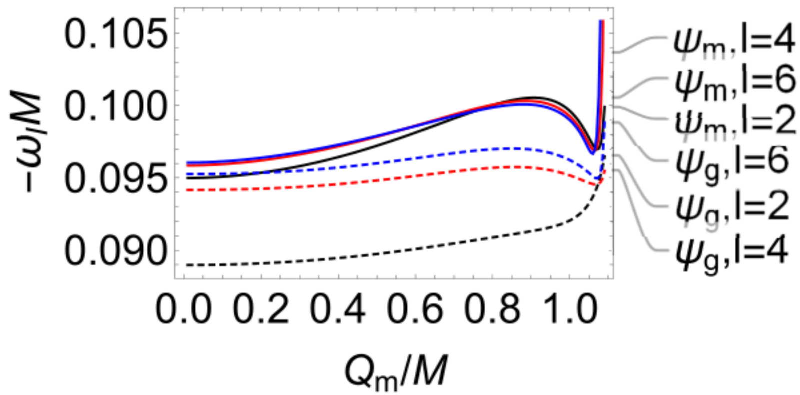


(a) The real parts of the QNFs for the charged black hole with scalar hair.

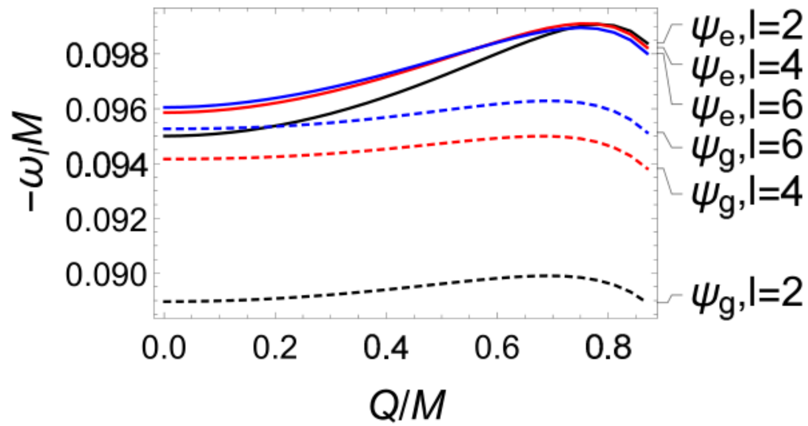


(b) The real parts of the QNFs for the RN black hole.

3. Quasinormal modes



(a) The imaginary parts of the QNFs for the charged black hole with scalar hair.



(b) The real parts of the QNFs for the RN black hole.

4. Conclusions

We studied the QNMs of this charged black hole by studying the linear perturbation of the gravitational field and the electromagnetic field.

We obtained two coupled perturbation equations.

The extra dimension radius Ry has no effect on the QNMs.

The differences of the frequencies of fundamental QNMs between the charged black hole and the RN balck hole are very small.

Thanks for your attention!